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PRACTICAL ILLUSTRATIONS

OF THE

ACHROMATIC TELESCOPE.

BEING THE SUBSTANCE OF TWO PAPERS READ TO THE
SOCIETY, 8TH NOVEMBER AND 13TH DECEMBER, 1836,

By ANDREW ROSS.

PART I.

WHEN a lens, which is convex on both sides, or plane on
one side and convex on the other, is held between a
candle and a distant sheet of paper, two positions will
easily be found, in which it will form upon the paper an
inverted picture of the candle; and the magnitudes of the
candle and its picture will in every case be exactly pro-
portional to their distances from the lens. Thus, when
the lens is twice as far from the candle as it is from the
paper, the dimensions of the candle will be double those
of the picture, and *vice versâ*.

Then, since the apparent magnitudes of objects vary
inversely as their distances from the eye, it is clear that
the candle and its picture would always appear of equal
magnitude to an eye placed at the centre of the lens. It
is equally clear, that if the eye were to approach nearer to
the paper it would see the picture under a greater angle,
and therefore larger, than, from the same point, the
candle itself would appear. It would therefore see a
magnified candle; and, though this would be a very
useless operation in a case where the eye could with

equal ease have approached the object itself, yet, if it be supposed that a picture had been thus formed of the sun, towards which the eye can make no sensible approach, it is evident that means are thus furnished of magnifying the sun or any other distant object, which are wholly dependent on the power of forming its picture by a lens.

To revert, however, to the candle, it will be observed that the picture is visible in all directions, or at least from every point in a hemisphere having the paper for its plane; and if the paper be transparent, it will really be visible in all directions, or from every point of a whole sphere having the picture in its centre. Then, by going to the back of the paper, the picture may be viewed and magnified by another lens, in the same way that small print or an insect is magnified by a common hand-glass.

The picture on the paper is, in short, precisely similar in these respects to an actual transparent painting of equal size and brilliancy; and its production is due to the circumstance, that the relative positions of the candle, the lens, and the paper, are such as to cause all the light which falls upon the lens from any one point of the candle, to converge into another corresponding point upon the paper.

Let it now be supposed that the paper is suddenly removed; this can make no change in the condition of the rays of light between the candle and the former place of the paper. The rays from any one point will still meet in the air, as they before met upon the paper; but, instead of being arrested there and diffused in all directions, they will pursue their several courses, now diverging from each other till they meet some fresh impediment, which they will, to a certain extent, illuminate, but on which they will form no picture.

In the optical sense of the term, however, the picture still exists; for, although it be invisible, it is reproduced to the eye whenever the paper is restored to its former place. This condition of the rays of light, in which they are ready to paint a picture of an object so soon as a surface is properly placed to receive them, is technically called forming an *image* of the object; and this term *image* will henceforth be employed to express this condition, because in the telescope the meeting of the rays is never rendered sensible by the interposition of such a screen as has been described. In the camera obscura and the magic lantern, on the contrary, the meeting of the rays is rendered visible by an opaque or transparent surface, and hence they are said to produce *pictures*.

The *image* may be shewn to exist after the removal of the paper in two ways. First, by placing the eye in the precise spot formerly occupied by the picture and looking at the lens, which will then appear entirely filled with a blaze of light; because the eye now receives the rays just at their convergence from every part of the glass: and, secondly, by receding a few inches from the former place of the picture, and looking through the lens towards the candle; when a reversed image of the latter will be seen, very much brighter than the picture formed upon the paper, and larger or smaller, according to its distance from the eye. This image is visible to but one person; unlike the picture on the paper, it is not diffused in all directions, and hence its superior brightness. It is visible only to an eye placed in some part of the line drawn through the candle and the centre of the lens; and to an eye so placed, it differs in no respect but in brightness from the picture on the paper viewed from the same point.

The image which has thus been proved to exist in space, may, like the picture on the paper, be magnified by the interposition of a second lens. Let the spectator apply to his eye such a lens to view the image as was before applied to view the picture, and he has, in all essential particulars, a refracting telescope, shewing a reversed image of the object. The first lens, by which the image is formed, has been termed the object-glass; and the second, which is applied to the eye, is called the eye-glass.

To the young student in optics, few things are more instructive than the repetition of experiments like these, which may be made at the most trifling cost. When the experiment is made, it will, however, be observed, that the image, whether seen through one or two lenses, is not quite so distinct and well defined as the object itself; and this will be more perceptible if, instead of a candle, the object consist of sharp black marks on a white ground. The figures on a watch-dial, for instance, will be much less clear and distinct when viewed in this way than when seen by the naked eye; and the lines of separation between black and white will be bordered with coloured fringes. The confusion thus produced will obliterate all markings below a certain size, while the larger forms will be rendered misty and obscure.

The obliteration, mistiness, and obscurity in question, all arise from the circumstance that no single lens can do what was supposed to be accomplished by that which formed the image of the candle. No single lens can bring all the rays which fall upon it from any one point, in a luminous or illuminated object, to any other point; hence a point in the object will be represented by a sensible space in the image or picture; and these spaces,

overlapping and interfering with each other, produce the mischief which has been described.

The precise nature of the defects of a single lens will be best understood by contrasting the actual progress of the rays through such a lens, with the case first supposed of accurate convergence to a point.

In fig. 1 of the engraving, the rays of light L L, &c. which are parallel, or diverging from one point in a very distant object, are supposed, after refraction, to meet precisely in another point, F, which is called their focus. This is the condition which would indicate a perfect object-glass; but, instead of the rays meeting at a single point, as F, they are subject to two different causes of error or aberration.

First, the surfaces of every lens, or at least one of them, must be spherical, because it is practically impossible to work other than plane and spherical figures with the requisite degree of truth.

Now it is demonstrable by calculation, and is abundantly proved by experiment, that no lens with spherical surfaces can refract the rays issuing from one point exactly to another point. They will be subject to the following, which is called *spherical* aberration.

Let the lines L L, &c. in figure 2, represent, as before, rays from a very distant point of light. Then those which fall near the margin of the lens will be refracted to a point as at F, while those which fall nearer to the centre of the lens will be refracted to a more distant point, as *f*; both points being, however, in the axis of the lens, if the rays L be parallel to the axis. The amount of error, or distance, F*f*, will be much less than is shewn in the figure, where it is purposely exaggerated to make it distinctly visible. Its quantity would, in fact, depend very much on the position of the lens. If, for example,

the position were reversed, so that the convex side were presented to the nearly parallel rays, the distance Ff would not be one-fourth of that produced by the position given in the figure, which has been chosen to justify, as far as possible, the exaggeration of the effect.

If a different form of lens were employed, as for instance one equally convex on both sides, but having the same focal length, the spherical aberration would be about one-fourth of that due to the form and position shewn in the figure.

Whatever be the amount, and however varied by circumstances, an error of this nature is inseparable from all spherical lenses; and no others, as before stated, are practicable. When the effect is measured along the axis, as from F to f, it is called the *longitudinal* spherical aberration; and this measure is generally employed for optical purposes. It is sometimes expressed by measuring the diameter of the smallest circle, by which a point of light can be represented, or into which the rays from a point of light can be brought, and this is called the *least circle* of spherical aberration.

The second source of error arises from the circumstance that, whatever be the form of the surface, the material itself acts upon different portions of each ray with different forces, and separates white light into an indefinite variety of colours.

This effect is exhibited in fig. 3, combined with the spherical aberration; and the upper central, and the lower marginal rays, are shewn dispersed and coloured after refraction, to render sensible to the eye the nature, but, of course, not the quantity, of the error in question. When the two errors are combined, the least circle of confusion, whose diameter is here represented by S S; is manifestly

greater than would be produced by spherical error alone.

But before any comparison can be made between the values of the errors thus introduced, each must be examined more in detail, and under simpler circumstances.

The dispersion of light into colours is not the result of any specific virtue in the lenticular form; it occurs whenever a ray of light passes from one medium into another more or less refractive, making any angle other than a right angle with the surface which separates the two media. In fig. 17, the ray of light L I, on penetrating the surface of the glass G G, will be not merely bent, it will be dispersed and coloured; and if glass of a more dispersive character be employed, the effect, as shewn in fig. 18, will be proportionably greater. In either case, the spreading would continue to increase until some change took place in the medium.

Such a change is shewn fig. 16, where the light L is incident on one side, F G, of the parallelogram of glass, D E F G; it continues to spread till it reaches the other surface, D E, where each colour separately emerges parallel to the original direction. But though separated and coloured, it is no longer divergent; and this parallelism of the emergent rays, M, to the incident ray, L, is due to the parallelism of the surfaces D E and F G of the glass. When the second surface of a piece of glass is not parallel to the first, as in figs. 14 and 15, and as was the case in fig. 3, the refractions at the second surface no longer undo those produced at the first, but the ray emerges into space, in a direction not parallel to that in which it was incident on the glass, and dispersed into colours which form certain angles with each other, an with the incident light, depending —

1. On the angle contained between the two surfaces of the glass, which are the angles E D F in figs. 14 and 15, and which, in the case of a lens, are the angles formed by tangents to its surfaces at the points of incidence and emergence.

2. On the refractive and dispersive powers of the glass.

3. On the angles at which the light is incident and emergent.

This latter condition is generally got rid of by comparing the effects when the light passes through a prism, parallel to the base, as in fig. 14.

Calculations which need not to be repeated here, if for no other reason than that they contribute very little towards making or understanding the telescope, prove that the dispersion of light into colours, as explained in fig. 14, spreads each point of light into a circle, dependent, in some measure, on the thickness of the lens, but whose diameter is, on an average for crown-glass, equal to one fifty-fifth part of the diameter of the lens; a proportion so great as to render it surprising that any thing could have been seen through telescopes made with such object-glasses. But although the coloured light from each point occupies this large space, it does so very unequally; for reasons which will be easily understood by again referring to fig. 14, where the letters R O Y G B I V indicate the parts of the prismatic spectrum, which are said to be coloured red, orange, yellow, green, blue, indigo, and violet respectively. Much more important information is, however, conveyed by the curve to the left of those letters, the ordinates to which, or the distances of which, from the straight line X X, represent the intensity of the light at the several parts of the spectrum opposite to such ordinates; and in the lower spectrum,

the attempt has been made to exhibit this difference pic-
torially. Thus it will be seen, that at least nine-tenths of
the dispersed light are contained between R and B; and
that there is a brilliancy in the yellow and orange which
renders even the red and blue comparatively feeble. For
the future, therefore, the indigo and violet light may be
altogether thrown out of consideration, as being too small
in quantity to affect any calculations made for practical
purposes. And this was done practically, long before it
was determined to be expedient by such observations
and reasonings as are here given; for it was found
that the best telescopes were those which left a certain
portion of outstanding, or uncorrected blue colour,
which was, in fact, the indigo and violet portion of the
spectrum.

Assuming, then, that all the light worth noticing is
contained between R and B, the diameter of the circle of
chromatic confusion would be diminished to about one-
hundredth of that of the lens. But, even in this smaller
space, the distribution of the light is very unequal; it is
much more intense in the centre than at the margin, as
may be thus roughly explained. It is evident, on in-
spection of fig. 3, that only red and blue light will spread
to the margin of the circle of confusion; that orange and
green will approach within a certain distance; and that
yellow will be confined to a comparatively small space
round the centre. The proportions of the circles thus
differently illuminated by the various colours, may be
supposed to be as figs. 9, 10, 11, 12, and 13; and the
result of their superposition may be as fig. 8, in which
the relative brightness of the centre will be due, not
merely to the number of superposed discs, but in much
greater degree to the superior intensity of the yellow and

orange light.   Fig. 8 will further serve to explain the
dilated and confused appearance of a star, or other small,
bright, circular object, when seen through an uncorrected
telescope; while fig. 6 will shew the effect which would be
produced by a good achromatic telescope.

In consequence of the unequal distribution within the
circle of chromatic confusion, its diameter might be safely
estimated at only one two-hundredth of that of the lens,
if it were necessary to make any such estimate ; which,
fortunately, it is not, as the practice will hereafter shew.
When in practice it is required to allude to the value of
the chromatic aberration, it is measured, like the spher-
ical, along the axis; and, for plate-glass, is about one
twenty-seventh of the focal length of the lens : and this
quantity, unlike that of the spherical error, is nearly
independent of form and position, so long as the same
focal length is retained.

Obvious as all this now appears, it escaped the notice
of Sir Isaac Newton, on whose discoveries all later pro-
ceedings are nevertheless based.   He took alarm at the
idea of representing a point by a circle equal in diameter
to one fifty-fifth part of the lens which formed it; and
having computed the spherical aberration to be some
hundred times less than this, he abandoned the idea of
refracting in favour of reflecting telescopes, which are inde-
pendent of the chromatic, though they retain the spherical
error, unless they are formed into parabolic curves.

It happens, however, that the distribution of the
light in the circle of spherical aberration is almost as
unfavourable as in that of the chromatic confusion it is
favourable.   If light of only one sort — as yellow, for
example — from a single point, were distributed by a
spherical lens into the smallest circle, it would present

an intensely bright ring, and a bright centre with a dark
intervening space, as described in fig. 7. This is obvi-
ously worse than a uniform distribution over the whole
space, and the consequence is, that spherical aberration,
instead of being some hundred times less injurious than
the chromatic, is possibly not ten times less. These
proportions are doubtless sufficiently vague on many
accounts, but, as before stated, they are not required in
practice ; and they are here introduced merely to give a
general notion of the difficulties to be overcome, and
which are obviated by processes not demanding any
accurate knowledge of their precise actual amount.

Before those processes are described, it may be well
to notice the earliest attempts to solve this important
problem. It had occupied the attention of other philo-
sophers as well as Sir Isaac Newton, and principally that
of Euler, who, in 1747, asserted the possibility of com-
bining various refracting substances so as to correct the
chromatic dispersion. The credit due to him for this
speculation is somewhat diminished by the circumstance
of its having been founded upon an erroneous hypothesis ;
and the same fact furnishes an apology for the elder
Dollond, who, on the faith of Sir Isaac Newton's con-
clusions, zealously denied the possibility of doing what
Euler proposed, but, nevertheless, commenced a series
of experiments, beginning with that which had led Sir
Isaac Newton to his unfavourable opinions, and which
ended in accomplishing all that Euler had declared, and
Newton had hoped, to be possible. These experiments,
which included the spherical as well as the chromatic
correction, were completed in the year 1757, and the glory
of achieving this most valuable result is in no respect les-
sened by the fact, of which there is now no doubt, that a

chromatic correction had been to some extent produced in the year 1733, by Mr. Hall, a private gentleman of Essex, who, however, had not made public his invention.

The name of Dollond has thus become deservedly and indissolubly associated with the achromatic telescope, as it is still called, notwithstanding attempts to substitute the term " aplanatic," meaning " without error," and which is, in some respects, more appropriate, inasmuch as a good instrument has the errors of sphericity balanced equally with those of colour.

While, however, the term employed is of little consequence, it may be useful to point out a false inference to which many are led from considering it as more than a term. It is sometimes supposed that the visible colour is the chief defect of the instrument. But the quantity of visible colour upon some objects may be a very imperfect test of achromatism ; and its visible presence, even when considerable, is not the worst consequence. Colour becomes very sensible only at the lines separating light and dark spaces. Less decidedly marked spots exhibit little colour, though they are equally rendered indistinct. The coloured margin of more decided spots is not only an evil in itself, but is doubly injurious, as it indicates the existence of chromatic dispersion in all the points (or what should be points) of which the image is composed. In looking at the moon, for instance, with the worst telescope, very little colour is seen on the moon's disc ; because the chromatic circles formed by the several points of the soft shadows overlap each other, and produce the effect of nearly white light. The destruction of colour, by this means, proves of itself how completely distinctness is sacrificed, as is evident when

an attempt is made by such a telescope to examine any
of the more delicate markings on the moon's face.   It
must be remembered, therefore, that in speaking of chro-
matic dispersion, it is not so much the *colour*, as the *size*
of the dispersed circle, which produces the ill effects
under consideration.

This point may be very simply illustrated by the
common experiment of taking a box with a pin-hole
in the centre of one end, and a piece of thin paper at the
other.   Such a box, held with the pin-hole opposite a
candle, will exhibit a picture of the candle on the paper
similar to, but fainter than, that produced by the lens in
the experiment first mentioned.   If the hole be extremely
small, the picture will be well defined; but it will be
proportionably faint, because the fineness of the hole
which limits the spread of the rays going to form each
point, diminishes the quantity of light in the same pro-
portion.   If the hole be enlarged to the size of a pea
for instance, much more light will be obtained, but the
picture will be nearly obliterated; because each point of
the object being now represented in the picture by a spot
at least equal in diameter to a pea, the images must over-
spread each other so much as to produce a mass of con-
fusion.   It will be evident, for example, that if the object
whose picture was required consisted of small spots, at
such distances that they would be separated on the pic-
ture any quantity less than the diameter of a pea, their
images will overlap each other and intermingle, so soon
as that which represents each point is made equal in
diameter to a pea.   In other words, they will become
undistinguishable in the picture; and thus it will be easily
understood why the minuteness of any space which an
optical instrument will define, becomes a test not merely

of its power, but of what is much more valuable, its distinctness, or, to speak technically, its *definition*.

It must have been noticed, that the confusion produced by the enlarged hole is quite independent of colour. The images formed by the pea-sized aperture would be as achromatic as those formed by the pin-hole: they are, in this respect, superior to the images formed by the lens in the first-mentioned experiment; yet the picture formed by the lens, if it were at all judiciously chosen, would be much more distinct, notwithstanding its chromatism, than that formed by the pea-sized hole. It is to be observed again, therefore, that the great mischief of the chromatic error is the *size*, and not the *colour*, of the spaces which, in the picture, become the representatives of points in the object.

Previous to entering upon the correction of these errors, it will be necessary to explain them in reference to concave lenses; by which is here meant, all lenses which are thinner in the centre than at the circumference, notwithstanding that one side may be convex. In fig. 4, the parallel rays L L, &c. are incident on the plane side of a concave lens, from which they would emerge, diverging in such manner that if the central emergent rays be produced backwards, as shewn by the dotted lines, they would intersect the axis at f, while a similar process applied to the marginal rays would produce an intersection at F. This is the spherical aberration of the concave lens in question; and though f F is no longer the distance between actual, but between imaginary foci, it is, nevertheless, a perfectly correct measure of the effect. In like manner, each emergent ray will be dispersed, and this dispersion is shewn upon the upper ray, where the letters B and R indicate the order of the colours; while the continuation of the

lines B and R to the axis, which they intersect on either
side of F, gives the measure of the longitudinal chromatic
aberration, which is still equal to about one twenty-
seventh part of the focal length of the lens : the exact
quantity being, as before stated, dependent on the dis-
persive power of the glass.

That the chromatic and spherical aberrations of a con-
cave lens are exactly equal, and contrary, to those in a
corresponding convex lens, is easily proved by imagining
the concave side to be filled up by a plano-convex lens,
as shewn by the dotted line. The two lenses would form
a plate of parallel glass, and as each separately produced
chromatic and spherical effects, from which the combina-
tion must be evidently exempt, it is obvious that their
errors mutually balance and destroy each other.

These errors, as before stated, are not the result of
any specific effect belonging to the lenticular form, as is
proved by causing a ray of light to traverse the centres
of any number of lenses, as in fig. 5, either in the direc-
tion of the axis, or inclined to it at any angle. When
the ray is in the axis, no effect at all is produced; and
when inclined to the axis, but still passing through the
centre of the lens, no sensible effect is produced, because,
although the ray is bent and dispersed at the *first* surface,
each colour meets the *second* surface at nearly the same
angle as that under which it had quitted the first, and
they therefore emerge nearly parallel.

This separation into parallel colours (as explained in
fig. 16) is insensible to the eye on two accounts. First,
the width of the parallel coloured beam M, produced,
under any common circumstances, by a single ray of
light, would be so much smaller than the diameter of
the pupil, that an eye would receive the whole of it, and

converging the whole to a point on the retina, would recompose it into white light.

Secondly, when the incident beam L has any sensible width, that width is composed of an indefinite number of rays, which would emerge as an indefinite number of coloured beams M; of which one would contribute red, another yellow, another blue light, to every part of the compound emergent beam, and which would, therefore, except at its extremities, be in fact white, independently of the faculty in the eye to recompose the various parallel colours into their original whiteness.

The faculty of recomposition by the eye is the best and most universal explanation of this phenomenon, and it may be thus generally stated. Whenever light is brought to the eye diverging from a point within the limits of distinct vision (say more than ten inches distant), or *parallel*, which is merely an expression for diverging from a point infinitely distant, so long will a well-defined image of a point be presented to the mind; and this will be quite independent of the colours in such divergent light, for though the colours may not be in such proportions as to produce a white point, the want of due proportion will merely colour the point without impairing its more important quality, namely, definition. This proposition will be hereafter found, in considering eye-pieces, to be full of most valuable consequences.

Having now described the effects of refraction and dispersion in general terms, their actual amounts must be investigated under simpler circumstances than in connexion with lenses or prisms.

Referring again to figure 17, G G is the surface of a piece of plate or crown-glass of indefinite thickness, upon

which a ray of light L is incident at I, and is refracted and dispersed, as indicated at R Y B. If the line P p, perpendicular to G G, be drawn through the point I, the angle L I P is called the angle of incidence; R I p is the angle of refraction for the red rays; Y I p the angle of refraction for yellow rays; and B I p that for blue rays. The line M N is the sine of the angle of incidence, and the lines R r, Y y, and B b, are the sines of the angles of refraction for the red, yellow, and blue rays respectively.

Now there is one most important fact connected with these sines, namely, that, however the angle of incidence, and consequently the sine M N, may change, the other sines R r, &c. will always change in exactly the same proportion as M N. The proportions between the sine of incidence and those of refraction being once established for any angle of incidence, they remain constant during every change of L I, from the perpendicular P I down to the horizontal G I.

The actual proportions for crown-glass of average quality are thus stated :—

If R r be called 1, M N will be 1·526
Y y ..   1, M N   ..   1·530
B b ..   1, M N   ..   1·536

And if the extreme violet rays had been included in the figure, their sine being called 1, the value of M N would have been 1·546.

Whoever has seen the prismatic spectrum, even under the most favourable circumstances, must be aware that the boundaries of the various colours are so insensible, that very nice measurements of the values of the sines based on this alone would be absurd. Since, however,

the discovery by Fraunhofer, of numerous fixed and well-defined lines in the spectrum, certain of these lines have been selected as the representatives of the different colours in which they are constantly found, and they give more perfect uniformity to such measurements than had been previously obtainable. As regards the colours themselves, they are, in fact, infinite; and the supposition that the green of the spectrum is formed by the combination of blue and yellow, merely because superposed spectra of blue and yellow will produce green, is perfectly gratuitous. If differences of colour result, as is most probably supposed from differences of rapidity in the vibrations which produce light, no idea can be formed but that of a constantly changing frequency of vibration; and, therefore, of an essentially infinite variety of colour along the whole of the line X X, in fig. 14, totally independent of superposition.

Leaving this question as somewhat beside the present purpose, it may be stated that the numbers above given, 1·526, 1·530, and 1·536, are called the indices of refraction for the red, yellow, and blue rays in crown-glass. When the refractive power of glass is described without reference to its dispersion, it has been usual to give the refractive index of the central point of the spectrum X X, fig. 14, which generally falls near the margin of the green light. This is called the *mean* refractive index; and for crown-glass, such as is supposed in figure 17, is about 1·533.

In fig. 18, G G is intended to represent the surface of a piece of flint-glass, whose dispersive power is much greater in proportion to its refractive power than that of crown-glass. All that has been said in reference to fig. 17 is equally true of fig. 18, and the actual

numbers or indices of refraction are on an average as
follows :—

Red rays	1·628
Yellow	1·635
Blue	1·648
Extreme Violet	1·671

On comparing the numbers in the two preceding
tables of refractions, it will be observed, that for crown-
glass the difference between red and blue is 1·536-1·526,
or ·010; while in flint-glass it is 1·648-1·628, or ·020.
Thus, while the dispersions differ as 2 to 1, the refractions
differ only as 1·648 to 1·536.   In other words, the dis-
persive power is not proportional to the refractive power,
as Sir Isaac Newton inferred from his imperfect experi-
ment, neither can it be reduced to any given law, as
Euler imagined.   The differences are found by experience
(so far, at least, as they are at present understood) to be
quite arbitrary.   It is certain that the dispersive power
has no assignable relation to the refractive power; for
though it has been seen, in the case of flint and crown-
glass, that a small increase of refraction was accompanied
by a large increase of dispersion, yet, in the case of the
diamond, a very large increase of refractive power is
accompanied by a relatively smaller increase of dispersion,
as compared with flint-glass, while the diamond has a
higher rate of dispersion, as compared with crown-glass.

The comparison of the dispersive powers of various
substances is made by dividing the total dispersion by the
decimal part of the index of refraction.   Thus the dis-
persive power of crown-glass, for rays between red and
blue, is

$$\frac{\cdot 010}{\cdot 531} = \cdot 0188,$$

while that of flint-glass will be

$$\frac{\cdot020}{\cdot638} = \cdot0314.$$

Thus, though the absolute dispersions were as two to one, the relative dispersions, compared with the refractions, are only as ·0314 to ·0188, or, in smaller numbers, as 5 to 3.

These proportions, like every thing which depends upon a discrimination between the shades of colour in the spectrum, must be considered only as approximations, unless they are obtained by means of the fixed lines before mentioned: they are, however, of little consequence, because no two specimens of glass would give precisely similar results; and because, further, as the practice will shew, no actual appeal to them is made during the construction of a telescope. It is, nevertheless, desirable, and indeed necessary, with a view to saving time and material, that such approximations should be familiar to the mind, as guides for making certain assumptions, on which the actual processes are to be based.

Presuming, then, that ·638 and ·531 represent with sufficient accuracy the refractive powers of flint and crown-glass, and that ·0314 and ·0188 are their dispersive powers, and considering that both descriptions of glass are liable to great differences, it will be sufficient to impress the mind with the average properties, by calling the refractive powers as 6 to 5, and the dispersive as 5 to 3.

These proportions equally obtain, when the ray is refracted by emission from a denser into a rarer medium, as shewn in figs. 3 and 16. Were it not so, light would not emerge parallel to its original direction from a piece of parallel glass, as explained in reference to fig. 16.

If the glass, fig. 16, be divided by a plane D F, it becomes two prisms, similar to figure 14, having their summits or refracting angles D and F opposed; and this conclusion is, of necessity, enforced, that two equal and similar prisms, thus placed, will each undo the refraction and dispersion of the other : for it may be here stated as an invariable law, that, whatever is true of light transmitted in one direction through any number of media, is equally true if it be supposed to move in the opposite direction. Thus, if the coloured beam M, in fig. 16, or the dispersed light R O Y, &c. in fig. 14, be considered as the incident, instead of the emergent light, the rays L and L in those figures will be correct representations of the mode in which such coloured light would emerge.

If, then, the prisms E D F and D F G will mutually correct each other's refraction and dispersion, whichever way the light is supposed to move ; and if D E F, fig. 15, be a prism of glass, whose dispersive power is double that of fig. 14, while its refractive power is only equal ; it will be certain that such a prism, whose refracting angle D is only half the similar angle in fig. 14, would, if applied properly to the first, undo the whole of the dispersion, but preserve half the deviation or bending. Supposing, in short, that the two spectra in figs. 14 and 15, produced by the incident light L and L in each, are, in their several colours, equal and parallel ; two such prisms would balance their dispersions, yet retain a deviation equal to the inclination of the rays L and L in the two figures.

In fig. 19, two pairs of such prisms are so placed, and, therefore, any light as L L, incident parallel to L in fig. 15, would emerge uncoloured, and parallel to the ray L in fig. 14.   A combination is thus furnished, which

bends without dispersing; and if this combination be
supposed to revolve round its axis A F, it would generate
a figure conical on one face and plane on the other; and
which would be divided by a conical surface into two
differing refractive media. By this revolution, each pair
of rays L L will represent cylindrical surfaces of light,
and the emergent rays meeting in F and f will become
conical surfaces of light. Thus the means are obtained of
bringing any number of parallel cylindrical surfaces into
as many parallel conical surfaces, whose apices will be
arranged along the line fF, and so on till they meet the
glass.

But instead of so many parallel cones, the deside-
ratum is, as in fig. 1, cones meeting in a common apex;
to effect which, it is necessary to have as many various
refracting angles as there are different cylinders of light
to be refracted to a common point. Suppose there are
three such cylinders, as indicated by the lines L L L L L L
in fig. 20, then there must be three pairs of conical
arrangements, as X Y Z; of which the surfaces in Y Y
may be supposed parallel to those in fig. 19; then the
cone of emergent light, Y Y F, will be also parallel to
the cones in fig. 19. The portions X X, having a greater
refracting angle, will bend the rays into a more obtuse-
angled cone; while the portions Z Z, having a smaller
refracting angle, will bend the light into a more acute
cone; and if the angles have been correctly adjusted, all
these cones will have their apices in F.

What is true of three combinations, is true of any
number; and if a still further subdivision be made of
the pieces X, Y, and Z, they at length are resolved into
an infinite number of such combinations, having an infi-
nite variety of refracting angles, and capable, therefore, of

bringing an infinite number of cylinders of light to a common point or focus, F, and this without any dispersion.

In this way, the infinite series of conical combinations becomes converted into a concave and convex lens, of which the curvatures are not known, but which may be deduced from calculation; it is certain that they are not spherical, because, by the whole train of reasoning, they are as free from spherical as from chromatic error. Thus has been proved the possibility of determining curves which should be independent of error from figure. Such curves have in fact been determined, and may be seen in most of the good treatises on optics; but as they are impracticable to the manufacturer, and as the present object is wholly practical, this part of the subject may be dismissed.

Compelled, therefore, to adopt spherical figures, the next step is to determine what spheres, or portions of spheres, will most nearly realise the conditions of these imaginary lenses, and afterwards correct the spherical error by means to be presently explained. It has been already shewn, that if the flint-glass has a dispersive power, as fig. 15, double that of the crown-glass, fig. 14, then the angle of the flint-prism will need to be only half that of the crown-prism, to produce a complete correction. In other words, the refracting angles of the two prisms must be inversely as their dispersive powers.

Let fig. 21 represent two pairs of such prisms thus adapted to correct each other's dispersion, then the two lenses most nearly fulfilling the same conditions would be such as had their focal lengths directly as their dispersive powers, because the focal length of a lens is very nearly in the inverse ratio of the refracting angles of the prisms drawn within it, as in fig. 21. The pair

of lenses drawn in outline round the prisms, would therefore form an approximate correction as to colour, while the incident light, L L, would emerge at M M, tending to a focus at such a distance as shall have been determined on.

The nature of the chromatic correction in the lenses is illustrated in fig. 22, where the two lenses have focal lengths in proportion to their dispersive powers; and the points P and P are those which coincide when the lenses are in contact. Then L and M are the rays of uncoloured light, the first incident parallel to the axis, and the second proceeding to the focus of the compound lens. If this latter be considered as an incident ray also, both these rays would emerge at P and P, dispersed and coloured so that P R and P B in each, or the red and blue rays in each emergent pencil, would be respectively parallel. Then, since it has been stated that what is true of light in one direction is equally true in the opposite direction, if it be supposed that the coloured beam which has been supposed to emerge from M P is incident on the concave lens at P, it would emerge a colourless ray at M. Now this supposition precisely describes the actual case. The coloured beam emerging from the convex lens has each colour parallel to the corresponding colours incident on the concave lens, and though they appear in a different order in the two coloured beams, this is not the case after they have passed the intersection a short distance from the concave surface; and this does not affect the demonstration, because it has been shewn that nothing is required but parallelism to produce perfect definition of a point.

The correcting power of this combination might be

shewn otherwise than by the parallelism of the several
coloured rays, thus :— Produce the coloured rays of the
two lenses respectively till they meet the common axis of
the lenses; then the lines so produced will be found to
embrace equal portions of the axis. This follows obviously
enough from the parallelism of the several colours, but it
is necessary to bear in mind that the correction for colour
will be equally perfect whether it be determined by
causing the coloured pencils of the two lenses to be
respectively parallel, or by ascertaining that they in-
tercept when produced, equal lengths on the axis. The
latter measure is of course the longitudinal chromatic
dispersion or aberration.

Under such circumstances the two lenses in question,
when placed in contact, would form an achromatic object-
glass. If the dispersive powers of the two glasses are
as 5 to 3, and if the focal length of the convex lens
be 30 inches, that of the concave, or, as it is called, the
correcting lens, must be 50 inches, and the resulting
compound focal length will be 75 inches, because

$$\frac{1}{30} - \frac{1}{50} = \frac{1}{75}.$$

As the focal length will now become an important
element in the calculations, the mode of deducing it
from the known curvatures and refractive power of any
glass lens must be explained. This is done for parallel
rays according to the formula

$$\left(\frac{1}{r} + \frac{1}{r'}\right) a = \frac{1}{f};$$

or in words, the sum of the reciprocals of the two radii

of curvature, multiplied by the decimal part of the index
of refraction, is equal to the reciprocal of the focal
length.

If the point from which the rays emanate is so near
that they are sensibly not parallel, we must introduce
the distance of that point represented by d in the formula
thus,—

$$\left(\frac{1}{r}+\frac{1}{r'}\right) a + \frac{1}{d} = \frac{1}{f}.$$

The focal length of a concave lens is computed in the
same manner.

In these formula, a was supposed to represent the
decimal part of the mean refractive index; but if for
this is substituted successively the refractive indices of
red, yellow, and blue rays, so many various focal lengths
will be obtained; and by thus estimating the dispersions
of two lenses, calculating the convex for parallel rays,
and the concave for rays proceeding from the compound
focus, so that the chromatic aberrations should be equal,
the result would give an achromatic combination.

This, however, would be a difficult mode, practically,
of making the correction, the theory of which is now
sufficiently explained; and if the dispersive powers of
the two glasses were in the same proportions for all
colours, the correction would be complete. But such is
not the case. It has been stated, that the absolute dis-
persive power of some specimens of flint-glass is nearly
twice that of certain kinds of crown-glass; but if this
proportion were quite correct for the mean ray, it would
be by no means correct for the other colours; some of
which will be dispersed by the flint-glass more than twice
as much as by the crown, and some less than twice as
much. Thus the ratios of dispersion between certain

specimens of flint and crown-glass, determined by Fraun-
hofer from the fixed lines before-mentioned, are set
down, for the five principal colours, as follows:—

Red .... 1·900
Orange . ... 1·956
Yellow .... 2·044
Green .... 2·047
Blue .... 2·145

This difference in the ratios, according to which two
different glasses disperse the various colours, is called
the irrationality of their spectra; and it is obvious that
the mode of correction by two glasses requires for its
completeness that these several ratios should be equal, or
that the irrationality should not exist.

In consequence of this, the most perfect telescope of
two glasses must have, in addition to the outstanding
violet and indigo light before described, a certain portion
of uncorrected colour, which may be detected in very
well-made telescopes as giving green and claret-coloured
fringes to the borders of bright objects. These colours
are called the secondary spectra; they are very faint,
and of small extent, and, except in instruments of ex-
ceedingly short focal length, as compared with their
diameters, do not sensibly impair their performance.
They may be got rid of to a certain extent by com-
binations of three different glasses instead of two, as is
commonly done for very short telescopes of large aper-
ture. But, for longer instruments, the damage produced
by the secondary spectra is found to be less than that
introduced by the increased number of surfaces in a
triple object-glass. It is demonstrable, nevertheless, that
the secondary spectra do impose limits on the aperture

which it is possible to give to telescopes; though the
difficulty of obtaining glass has hitherto prevented those
limits from being reached.

Without, therefore, entering into the calculations in-
volved in this question, it will be well to shew the mode
of uniting with the chromatic correction, so far as it has
been obtained, a nearly perfect correction of the spherical
aberration of the two lenses.

The spherical aberrations of concave and convex lenses
are in opposite directions, as is evident by comparing
figs. 2 and 4. They, therefore, to some extent, always
correct each other; but a perfect balance would be a
mere accident, inasmuch as this has not been at all con-
sidered in correcting the colour.

It has been already stated that the dispersion of a
given lens is always proportional to its focal length, and
that it remains constant however the form of the lens may
change, provided that no alteration be made in the focal
length. Spherical aberration, on the contrary, depends
more upon the form, or the ratio of the radii, than upon
the focal length. It is possible, therefore, to make an
indefinite amount of change in this ratio, and of conse-
quence in the spherical aberration, without disturbing the
chromatic correction which depends on the focal lengths
alone.

The effect of spherical aberration, when it exists sen-
sibly, is to envelope any small brilliant object in a colour-
less mist, whose extent is, of course, proportioned to the
amount of error. To correct this, it is necessary to revert
to the formula for the focal length of a lens,

$$\left(\frac{1}{r}+\frac{1}{r'}\right) a = \frac{1}{f},$$

in which it is evident that $\frac{1}{r}$ and $\frac{1}{r'}$ may both vary from

0 to their sum, one increasing as the other diminishes without altering the result, and, therefore, without affecting the focal length.

In illustration of this, fig. 23 exhibits seven convex lenses having equal focal lengths, and, therefore, equal chromatic errors, but having spherical aberrations varying from 1·05 in the best, to 7·00 in the worst, form and position. The whole of this immense difference depends, first, on the ratio of the radii; and, secondly, on the side which is opposed to the incident light. The upper figures attached to each lens give the ratio of their radii, and the lower figures the amount of spherical aberration due to each.

In the first, which is plano-convex, opposing the convex side to the parallel rays, the ratio of the radii is as 1 to infinity; and this being a favourable form and position for parallel rays, the spherical aberration is only 1·05.

In the second, the ratio of the radii being as 1 to 4, and the focal length as before, the value of the spherical aberration is increased to 1·1.

In the fourth, which is equally convex, or having the ratio 1 to 1, the aberration is 1·7.

In the last, the ratio of the radii is like the first; but in the reverse order, and hence the aberration is increased to 7·0.

It is easy, therefore, to understand, that if the achromatic combination shewn in fig. 22 have any spherical error, it may be destroyed by altering the ratio of the radii of the convex lens. To do this, first ascertain correctly the spherical aberration of the concave lens for rays proceeding from the focus of the compound lens; and supposing it to be 1·73, fig. 23 shews that a ratio of equality

gives 1·7 : which is near, but not sufficiently near, to what is required.

Calculations must, therefore, be made similar to those by which the aberrations here set down have been computed, and for which the formulæ are well given in Wood's *Optics*. From these it may be deduced, that the ratio of radii which will produce the required aberration is 21·7 to 20·5. Make r and r' in this proportion without altering the sum of their reciprocals, and such a lens will balance the spherical aberration of the concave ; the focal length of the latter having been previously determined to be such as to correct the colour of the convex.

The actual radii of an object-glass recently constructed on this plan with success, were as follows : —

> First surface of convex .... 21·7
> Contact   do.   .... 20·5
> Contact of concave .... 20·5
> Second surface of do. .... 205·0

These numbers may suggest the inquiry, how it happens that the spherical aberrations of a nearly equally convex lens are balanced by those of a concave having radii so widely different, — a result which would not seem probable from an inspection of fig. 23. Nor would such a result obtain if the light were incident on the two lenses under the same circumstances ; the force of which observation will instantly occur to the mind, on looking at the different angles at which the light is incident on the first and second lens in fig. 22. In calculating the spherical aberration of the concave lens, it must not be taken for parallel rays, but, as before stated, for rays like M, fig. 22, proceeding from the compound focal point ; and hence the value of the proposition that light is subject to the

same effects, in which ever way it is supposed to move. The direction M, depending on the focal length, is a certain and easily obtained element for calculation; but the direction of the coloured beam between the lenses would be both difficult and laborious to determine.

The importance of this consideration in estimating the aberration of the concave lens is shewn by the fact, that while its error for rays, under the given circumstances, is only 1·73, its error for parallel rays incident on the plane side would be 3·5.

It may have been noticed, that in the dimensions of the given object-glass the radii of the contact surfaces were equal. This is not essential to the correctness of the combination, nor does it ever happen unless specially aimed at in the calculations, which are thereby rendered much more complicated and difficult. In this case, as in many others, it was done for the sake of cementing together the contact surfaces, which could not be done unless their radii were equal, as otherwise the cementing medium would form a third lens.

Those who would enter minutely into the correction of spherical aberration, having reference both to the ratio of radii and direction of the incident pencil, may study the subject geometrically in Wood's *Optics*, and in an elegant analytical style in Coddington's works.

PART II.

HAVING now explained generally the theory of achromatic, or rather aplanatic, object-glasses, I shall proceed to describe the mode of applying the theory by detailing

my own practice, in reference to two specimens of glass whose optical properties I presume to be quite unknown.* I shall, however, premise a short account of the state of the art of telescope making, after it had received its last improvements from the genius of the elder and the younger Dollond.

No sooner had these able artists realised experimentally the propositions of Newton and Euler, than the subject began to occupy the attention of the most eminent mathematicians in Europe. The Transactions of the foreign academies were crowded with memoirs on the determination of the radii of curvature for achromatic glasses, and as the question afforded full scope for the deepest geometrical and analytical research, it occupied the labours of Clairaùlt, D'Alembert, Boscovich, and many other illustrious names.

Unfortunately the calculations of these profound mathematicians were sometimes based upon erroneous assumptions, and were always of a nature to be little understood by the practical opticians who should have tested their results by experiment. The effect of this was to produce in the minds of practical men a distrust of, and, perhaps, some degree of contempt for, modes of inquiry which they were unable to appreciate, and which too often justified that distrust by the errors they involved.

In England, at this period, the cultivation of mathematical science was comparatively neglected. The subject of achromatism was discussed in Martin's *Elements of Optics*, published in 1751, and was reduced to much apparent simplicity; but the principles were wholly

* For this part of the subject, I have found it more convenient to write in the first person.

erroneous, as may be well supposed from two examples.
The spherical aberrations of both lenses were computed
for parallel rays, and the same indices of refraction were
used for both sorts of glass.

The article " Telescope," contributed by Professor
Robison to the *Encyclopædia Britannica*, is a much
better treatise on the subject, but the methods are ex-
cessively difficult and circuitous ; so much so, indeed, as
to be nearly impracticable. I am not aware that any tele-
scopes have been actually constructed by the processes
there described, although no doubt has ever existed as to
their accuracy; while the simplicity of Martin's plans,
notwithstanding their ascertained errors, caused their
adoption by the late Mr. Tulley, who, however, made an
approximate correction as to the spherical aberration of
the concave lens, by using an empirical formula, into
which he introduced the dispersive ratios of the two
glasses and the refractive index of the flint. This formula
was obtained by comparing Martin's numbers with va-
rious carefully made telescopes. But as it was founded
on comparisons with object-glasses actually made, it was
of necessity limited in its application, and especially when
entirely new circumstances were presented. Mr. Tulley
further improved the practice by ascertaining the cor-
rect index of refraction of the crown-glass, Martin having,
as before stated, always employed the same numbers
for both.

In 1821 appeared Sir John Herschel's paper upon
" Aplanatic Object-glasses," than which nothing can be
desired more accurate or elegant ; and though, unfor-
tunately, it involves some practical difficulties, they are
such as have been subsequently removed only by the
sacrifice of that perfect beauty in the analytical inves-

tigations which the materials, no less than the persons
to be employed, render unavoidable. Sir John Herschel's
method gives to an object-glass the form shewn in
fig. 24, the outside of the flint-glass being convex, and
the contact surface being of greater radius than that
of the crown. This latter effect causes the surfaces to
ride in the centre, which is a mere practical inconve-
nience. The principal difficulty arises from the fact,
that the ratio of curvatures is such as to bring them
to the lower part of the scale in fig. 23, where it has
been shewn that every successive change in the ratio
produces greater and greater errors as we descend. Any
discrepancy, therefore, between the working-tools and the
computed curves, or any mistake in the computations
themselves, produces a much greater amount of error
than if the ratio of the radii fell nearer the upper part
of the scale. To Sir John Herschel, however, we are
indebted for numerous valuable suggestions, and for one
of singular beauty and simplicity, by which we obtain
the relative dispersive powers of the flint and crown-
glass, which will be explained when describing the pro-
cess of making a telescope.

In Coddington's *System of Optics* the question is
treated as might be expected from the high reputation of
the author; but his methods demand a familiarity with
the use of the differential calculus, which is scarcely
attainable by the working optician, and which certainly
could not, with convenience, be resorted to in each
particular case.

A consideration of these difficulties induced Professor
Barlow, in 1826, to undertake an investigation of the
whole matter, with a view to devise methods, which,
though they should be deficient in those qualities prin-

cipally admired by the pure mathematician, might combine, with perfect practical truth, such facilities as would recommend them to the moderately well-educated workman. I had the honour and the great advantage of making, for Professor Barlow, some of the experiments by which he arrived at his results: merely, however, as his agent, and under his instructions.

These results were embodied in a paper read to the Royal Society in 1827. They comprise the best and most available methods which have yet been made public; and they, with very slight modifications, form the basis of my own practice, which I shall now describe in detail.

Having two pieces of glass, the flint is easily distinguished from the crown by its superior specific gravity, arising from the quantity of lead which enters into its composition. Thus the glass which is called flint-glass, has in fact a smaller proportion of silica than the crown. It obtained this title from the care employed in selecting the flint with a view to the purity of the glass; and the term has since become so extensively adopted as to be now almost European.

The great specific gravity of the lead, and the consequent difficulty of keeping it suspended in the other materials during their state of fusion, has presented, for many years, one of the most formidable obstacles to the improvement of practical optics. Discs of homogeneous flint-glass, more than four or five inches in diameter, are exceedingly rare, and, of consequence, very costly. At present almost the only good specimens are obtained in Switzerland, from the widow of Guinand, who devoted a life to this object. Even these require a very close examination to detect inequalities in the distribution of

the lead, which produce veins and striæ, and which are quite fatal. Minor imperfections, such as specks and bubbles, may exist to some extent, both in the crown and flint-glass; but to detect them, as well as the veins and striæ, it is necessary, in the first place, to polish the edge of each disc on two opposite sides so as to look through the entire diameter.

Having selected two specimens which appear sufficiently free from all defects, they must be formed into lenses for the purposes of ascertaining their optical properties. In assuming curves for these experimental lenses, the practised judgment of the optician is employed to choose such as are likely to approach those which may be ultimately determined. His decision in this particular is based upon the specific gravities of the glasses, upon his knowledge of their manufacture, and, most of all, upon his experience of telescopes made from specimens nearly resembling them. Unless these assumptions are judiciously made, he incurs the certainty of some useless labour; and the risk, if the specimen of the glass be thin, of reducing it so much in successive trials, as to render it useless: a danger which applies eminently to the flint-glass, whose thickness in the centre must remain sufficient to support its figure in working.

On this account, the assumed radii of curvature are taken longer than those which are expected to result from the calculations; and the crown is assumed relatively longer than the flint, for the sake of obtaining the ratio of the dispersive powers by the very ingenious method of Sir John Herschel before alluded to.

Suppose, then, that the glasses are formed into lenses, whose radii of curvature are of course known. The first step is to ascertain the index of refraction of the crown

or convex lens, which is done in the following manner.
Mount the lens in a telescope tube in the usual way;
and in the focus of the eye-piece, which may be either
a micrometer or an erecting eye-piece, let a fine wire or
hair be stretched across the field. At some convenient
distance place a black board, having a white circular
disc with a black line across it. Direct the telescope
to this disc, and adjust the tubes till the image formed
by the lens is in the same plane as the hair, which is
easily ascertained by slightly moving the eye up and
down: for when the hair and the plane of the image
are not coincident, an apparent movement of the hair
upon the disc will accompany the movements of the eye;
but when the coincidence has been obtained, the hair,
and the image of the black line upon the disc, will be
inseparable.

Then measure, with great accuracy, the distance be-
tween the centre of the lens and the hair, which may
be called φ; also between the centre of the lens and the
white disc, which call δ. Introduce these quantities,
into the following formula :—

$$\frac{1}{\varphi} + \frac{1}{\delta} = \frac{1}{f},$$

f being the focal length of the experimental crown lens
for parallel rays; then r_o and r_c being the known radii
of curvature.

$$\left(\frac{1}{r_o} + \frac{1}{r_c}\right) a = \frac{1}{f} \text{ or } a = \frac{\frac{1}{f}}{\frac{1}{r_o} + \frac{1}{r_c}},$$

a being the decimal part of the index of refraction for
the glass in question; and, by adding 1, the refractive
index, $1 + a$, is obtained.

It may be as well here to state, that the decimal part of the index of refraction means the whole index, minus 1 : so that if the refractive index exceeded 2, the decimal part would consist of a whole number and certain decimals. In the diamond, for example, whose index is about 2·5, the decimal part is 1·5. No glass has yet been made, of which the refractive index has been so much as 2.

The next step is to obtain the index of refraction of the flint-glass, which is done by placing the concave lens in contact with the convex; mounting both in the telescope tube, and then ascertaining the focal length of the combined lenses, as before explained, for the convex. The measured distance of the disc is again called δ, and the distance of the hair, or focal length of the combined lenses for the distance δ, is called φ''; and thence f'', the focal length for parallel rays, is deduced as before,—

$$\frac{1}{\varphi''} + \frac{1}{\delta} = \frac{1}{f''}.$$

Then, calling the focal length of the concave lens, f', the value of this is obtained thus,—

$$\frac{1}{f'} = \frac{1}{f} - \frac{1}{f''},$$

and the refractive index of the flint-glass (a') thus,—

$$a' = \frac{\dfrac{1}{f'}}{\dfrac{1}{r'_o} + \dfrac{1}{r'_c}},$$

where r'_o and r'_c are the radii of the outside and contact surfaces of the experimental concave lens, which, like the others, are known, having been assumed.

It has been already stated, in reference to fig. 22, that to obtain equal dispersions by a concave and convex lens, or equal lengths of spectra when measured upon their

common axis, their focal lengths must be proportional to their dispersive powers. The focal lengths of the experimental lenses having now been ascertained, we next demand to know how nearly the ratio of those lengths corresponds with that of the two dispersive powers. By mounting them in the tube, and examining the edges of the black and white marks on the disc, we might easily determine whether the dispersion had by chance been corrected; but if not, how are we to learn the precise amount of error? This is the question which is answered by the very perfect and simple operation suggested by Sir John Herschel.

Mount the two lenses in such a cell as will permit them to take any position, from that of close contact to a certain moderate quantity of separation; say, for example, about equal to one-twentieth part of the focal length of the crown. Place the cell in the telescope tube, and then, if the focal length of the crown lens has been assumed relatively longer than that of the flint, the margin of the image of the white disc will be coloured; and it will become less so as the lenses in the cell are separated. A certain position will, at length, be found, which will give a colourless image; but it must be observed, that, as the lenses are separated, the telescope tube will require to be shortened.

We have now the means of determining with great accuracy the relative dispersive powers of the two specimens of glass. Suppose fig. 25 to represent the separated lenses in the state which produced the colourless image. Let the quantity of separation, or A D, be most scrupulously measured, from the centre of one lens to that of the other. Now, it is evident that, when the lenses were in contact, the rays incident on the concave surface

were converging to a distance Af; f being the focus of the convex lens. But, after the separation, the rays at the moment of incidence were converging to a point, whose distance from the concave lens is only Df, which is the same thing as if they had been refracted by a convex lens in contact with the concave, whose focal length was only Df. Such an imaginary lens is shewn in contact by the dotted lines; but while the separation has produced the effect of shorter focal length in the convex lens, it has evidently not diminished the longitudinal chromatic aberration which still remains, such as is due to the real lens or to a focal length Af.

Since, therefore, the ratios of the dispersions and focal lengths are equal when two lenses are achromatic and in contact, and if that ratio be called $s : 1 = f : f'$, then, in the other case, if the measured distance of the separated lenses be called d, the value of s must be obtained thus:—

$$s\,\frac{f}{f-d} : 1 :: f-d : f';$$

and this equated and transposed gives

$$s = \frac{(f-d)^2}{f'f}.$$

Let now the determined focal length of the intended telescope, which is, of course, a known quantity, be called F''; and let the requisite focal length of the flint lens be called F'; and that of the crown, F.

These two last quantities are not yet known; but they are obviously deducible thus:—

$$F = (1-s)\,F'',$$
$$F' = \frac{1-s}{s}\,F'' = \frac{F}{s};$$

and nothing is now required but to determine the radii of curvature by which these focal lengths shall be obtained.

To do this, a ratio for the radii of the flint lens is assumed; and it is in making this assumption, rather than deducing it from calculation, that Professor Barlow has consented to sacrifice mathematical elegance to practicability. Here the practised judgment of the optician is again required, because, if the ratio he assumes be injudicious, there is no mode of correction but making a fresh supposition, and recommencing the calculations from this point.

Let, then, this assumption be made, and let the ratio, or the outer radius, divided by the contact radius, be called q'. Then, if R'_o and R'_c be put to represent the radii of the outside and contact surfaces, their values will be thus found :—

$$R'_o = (1 + q')\, F'\, a',$$
$$R'_c = \frac{(1 + q')}{q'}\, F'\, a'.$$

Having now the focal length, the ratio of the radii, and the actual radii of the corrected concave lens, we can obtain its longitudinal spherical aberration from the following formula :—

But first let $\dfrac{a'}{1 + a'}$ be called b;

$\dfrac{f'}{R'_c}$ be called c;

And $\dfrac{(1 + a')\, F''\, q'}{a'\, F'' - R'_o}$ be called c',

then the longitudinal spherical aberration of the flint lens being called p,

$$p = \left[\frac{(c + q')^2}{(a'c - q')^2} \cdot \frac{c + (a' + 2)\, q'}{c\,(a'c + a' + 1)^2} + \frac{(c' + 1)^2}{(bc' + 1)^2} \cdot \frac{(c' + 2 + b)\, q'}{c} \right] \frac{a\,s}{q' + 1}.$$

This quantity represents also, of course, the amount of spherical aberration which must be given to the convex

lens, in order that it may balance that of the concave;
and the requisite ratio of the radii to produce it might be
calculated in the usual mode of calculating spherical
aberrations. This, however, would be an exceedingly
laborious operation; and here, therefore, Professor Bar-
low has again laid the practical optician under the
deepest obligations, by having made these calculations
for a variety of cases, and by having reduced them to a
tabular form, which may be easily referred to and applied.
The table is annexed; the columns headed p are the values
of the computed spherical aberration for the concave lens,
or, which is the same thing, the required aberration for
the convex lens. The figures in the columns headed q
shew the ratio of radii, which, in the convex lens, will
produce the aberrations required; always bearing in mind,
that the aberrations of the convex lens are calculated for
parallel rays, while those of the concave were computed
for rays diverging from the focus of the compound lens.

p	q	p	q	p	q	p	q
1·15	·38	1·40	·68	1·65	·98	1·90	1·29
1·20	·44	1·45	·74	1·70	1·04	1·95	1·36
1·25	·50	1·50	·80	1·75	1·10	2·00	1·43
1·30	·56	1·55	·86	1·80	1·16	2·05	1·50
1·35	·62	1·60	·92	1·85	1·22	2·10	1·58

To take an example, let it be supposed that p had
been found to be 1·60, then, opposite to 1·60, in the
column p, is seen, in the column q, ·92, which is the
required ratio of the radii in the convex lens. And if p
had been a quantity not given in the table, as, for

instance, 1·57, the corresponding ratio, or value of q, is easily interpolated, and would be nearly ·884.

Having got this, the following formulæ give the required radii for the outside and contact surfaces of the crown lens : —

$$R_o = (1 + q)\, Fa,$$
$$R_c = \frac{(1 + q)}{q}\, Fa.$$

Thus, by a method correct in principle, though somewhat indirect, and which requires only a knowledge of ordinary algebraic processes, may be deduced—

1. The focal lengths of the two lenses, for producing equal and contrary dispersions of the coloured rays; and, therefore, an achromatic combination.

2. The four radii of curvature, which will give those focal lengths, and, at the same time, produce equal and contrary spherical aberrations.

An object-glass thus constructed will, as regards the calculations, be as perfect as the present state of optical science can render it. No calculations, however, can dispense with that constant practical and skilful attention to the state of the working-tools; to the accumulation of the abrading material under the glass in the act of grinding; to the liability to lose the figure by unequal polishing; to correctness of centering; and to all those innumerable difficulties which are known to attend the working and adjustment of very large object-glasses for telescopes, and very small object-glasses for microscopes. On these points I am engaged in a series of experiments, with a view to render the processes in question more certain in their results, and less dependent on the hand of the workman.

By introducing a greater degree of certainty into these

operations, we shall be able to do more justice than has hitherto been done to the labours of those mathematicians who have investigated these subjects, and whose deductions have on some occasions, perhaps, appeared to be in error, only because of the imperfect mechanical means employed to realise them.

Having completed the object-glass, a very few words will explain its operations in connexion with the eye-glass. In fig. 26 is shewn the simplest form of eye-piece, consisting of a single lens with the plane side next the eye. The object-glass having formed an inverted image of the arrow, the eye-glass is so placed that the image is nearly in its principal focus; the rays of light after refraction by the eye-glass, therefore, come to the eye from each point in nearly parallel lines.

The magnifying power of this arrangement depends, of course, on the focal lengths of the object-glass and eye-glass; being greater as the former is long, and the latter is short. Suppose, for example, that the object is 1000 feet distant, and the focal length of the object-glass 10 feet, the image will be $\frac{10}{1000}$, or one-hundredth part of the real size of the object. It was stated in the outset, that an object and its image appear of equal magnitudes to an eye placed at the object-glass; but, as the object-glass is here ten feet from the image, and as the eye can see within ten inches, it will, by thus approaching the image, see it twelve times larger than it could see the object.

Suppose, further, that the eye-piece is a lens of one inch focal length, which will enable the eye to see distinctly within an inch of the image; then the magnifying power will be obviously increased tenfold by this still nearer approach, and will be $12 \times 10 = 120$.

An eye-piece of this description shews the inverted image produced by the object-glass. Eye-pieces have, therefore, been contrived, and are explained in fig. 27, by which a restoration of the erect position is effected, and which requires no explanation beyond that given by the course of the rays in the engraving.

Neither of these eye-pieces is, however, sufficiently good in its optical properties for the higher class of telescopes. The eye-piece invented by Huyghens, and shewn in fig. 28, where the rays are refracted at two lenses, yet without altering the position of the image, is greatly superior on two accounts.

It considerably increases the field of view of the telescope, and, what is of more importance, it to a certain extent corrects the colour which would be produced by refraction through a single lens. The principle on which this is effected was first popularly explained in the *Transactions of the Society of Arts*, by Mr. Cornelius Varley, principally in reference to the microscope. The corrections produced by this eye-piece are, however, equally important in regard to the telescope, where, on account of the smaller pencil of light, the chromatic aberrations are much more prejudicial than the spherical.

The annexed figure explains the action of this eye-piece. The plano-convex lens F F is called the field-glass, and E E is the eye-glass. The lines L, M, and N, indicate rays converging from the object-glass which would meet at B R, B R, and there form the image of the object. Two images are, however, shewn in the figure, for this reason : it is evident from fig. 3 of the engraving, and was, in fact, fully explained, that a single lens brings the blue rays to a focus at a shorter distance

than the red, and so on for the intermediate colours. A single object-glass here, therefore, would have brought
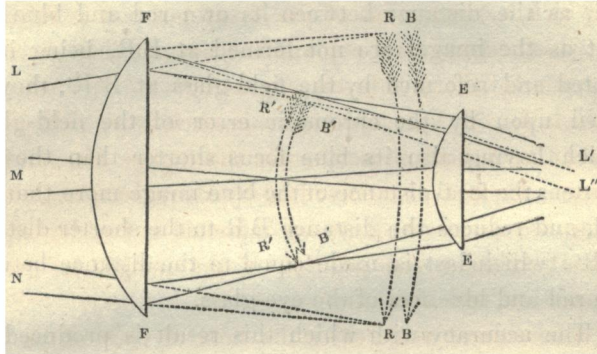
the arrow marked B B, intended for the blue, nearer than that marked R R intended for red; whereas in the figure, the blue arrow is most distant from the object-glass.

This effect is produced in the following manner. The whole explanation of the theory and practice of achromatism has shewn that we can combine two lenses of crown and flint-glass so as to bring the blue and red foci together, instead of permitting the former to fall short of the latter. But it is clear that the same processes carried a little further would reverse the order, and cause the red focus to fall short of the blue, the order of the intermediate colours being inverted in the same way. This is actually done in every well-made telescope : it is termed over-correcting the object-glass as to colour, and hence the relative positions of the red and blue arrows in the figure.

The reason for doing this is, that the eye-lens not

being achromatic has its blue focus shorter than the red;
it requires, therefore, for distinct vision, that the blue
image should be brought as much nearer to it than the
red, as the distance between its own red and blue foci.
But as the images are not formed at B R, being inter-
cepted and reformed by the field-glass at B′ R′, they are
acted upon by the chromatic error of the field-glass;
which having also its blue focus shorter than the red,
shortens the focal distance of the blue image more than the
red, and reduces the distance B R to the shorter distance
B′ R′: which last is made equal to the distance between
the red and blue foci of the eye-glass.

The accuracy with which this result is produced de-
pends, of course, on the accuracy with which the amount
of over-correction, or the distance B R, is determined.
These ultimate refinements are made experimentally, by
observing with the object-glass, and its intended eye-
pieces, minute reflections of the sun's disc; judging of
the state of the corrections by the eye, and finally alter-
ing the curves according to such observations.

We have not yet, however, completed the theory of
the eye-piece. It will be observed that the images
B R, B R, formed by the object-glass, are curved; they
are, in fact, spherical, having the object-glass for their
centre. This form is just the reverse of what the eye-lens
requires, because any lens so used demands, for a good
field of view, that the object or image seen through it
should be on a spherical surface, concave towards the lens.
Fortunately the field-glass changes the form of the image
so as, in great measure, to produce this effect. It, to a
certain extent, reverses the curvature of the image, and
in so far renders it better adapted to the eye-glass.

The field-glass produces another and still more bene-
ficial effect, of a nature quite different to either of those
before mentioned. The fact of the blue image being
nearer to the eye-glass than the red, would be of little
value, if the blue light from any one point fell on the
same part of the eye-glass as the red rays from the same
point. If they did so, the greater action of the eye-lens
upon the blue ray would refract it more than the red, and
they would proceed diverging to the eye, giving a coloured
and confused impression. But as an effect exactly similar
has been already produced by the field-glass, and which
is indicated by the smaller size given to the blue arrow
B′ B′ as compared with R′ R′, the blue rays, as shewn
by the dotted lines, fall nearer the centre of the eye-
lens than the red rays. Thus the red rays, which are
least refrangible in themselves, fall nearer the margin,
which is the most powerfully refracting part of the
lens, as was explained in fig. 2 of the engraving. This
is exhibited in the figure with reference to the rays
L, which, after passing the field-lens, become divided,
one portion (the red) being represented by *whole*, and
the other (the blue rays) by *dotted* lines. By tracing
these to the eye-lens, the nature of the whole correction
will be obvious; it will be seen at once that the spherical
error of the eye-lens balances to a certain extent (of course
not mathematically) the chromatic error of the field-lens,
and, of consequence, the red ray L′, and the blue ray L″,
emerge sensibly parallel.

Hence will be seen the importance of the fact explained
in reference to fig. 16 of the engraving, that to produce
a perfect impression of a point, it is not required to bring
all the rays which emanate from it into one line, but
merely to render them parallel.

Before I quit the subject of eye-pieces, it may be well
to mention that experiments have been recently made to
ascertain the effect of using for an eye-piece an achromatic
combination of crown and flint-glass, similar to the object-
glass, but of short focal length. The experiments have
been made with the lower powers of microscopic object-
glasses, and the results have appeared so far very satis-
factory: the field is of course diminished, but it seems
most likely that some advantages will result from having
a combination whose achromatism can be determined,
instead of one whose achromatism is, in great measure,
accidental; for it was wholly unforeseen by the inventor
Huyghens, and was only pointed out at a subsequent and
distant period by Boscovich.

It remains to notice the eye-piece which is used for
telescopes intended to contain micrometers, and which
is therefore called the micrometer eye-piece. It was in-
vented by Ramsden, and is shewn in fig. 29. The object-
glass forms its image immediately in front of what may
be still called the field glass, though its position is re-
versed, having now the plane side next the object-glass.
This image is seen with tolerable distinctness, as shewn
by the course of the rays; but not so well as by the
Huyghenean eye-piece. Its advantage, however, consists
in this: that if a minutely divided glass scale, or fine wires,
be placed exactly where the image is formed, the scale
and the image are magnified together; and the latter can
be measured, or a particular part of it can be indicated,
with great precision.

This arrangement involves no correction for colour or
spherical aberration : in all other respects it is superior to
the Huyghenean. I have reason to believe, that the most
perfect eye-piece, both for telescopes and microscopes,

would consist of this—Ramsden's arrangement—but composed of two aplanatic combinations. On this point, also, I am engaged in a series of experiments, which I hope shortly to publish.

ANDREW ROSS.

Regent Street, January 1840.